The sequence of cash flow in bankruptcy prediction: evidence from Poland

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Introduction

Much less interest in the literature has been dedicated to modeling corporate bankruptcy with cash flow rather than to financial ratios based on traditional „accrual” reporting („accrual” ratios), probably because cash flow is less useful, since the accounting department had created a cash flow statement a comparatively short time before. Another reason could be the non-uniform research outcomes on cash flow advantages in corporate bankruptcy prediction. Most research has seen little usefulness of cash flow in financial ratios, although a few studies have found it useful (Altman, 1968; Casey, Bartczak, 1985; Sharma, 2001).

In general, cash flow can be used in a bankruptcy model in two ways:

• as a part of financial ratios,
• as a relationship of items in a cash flow statement.

Although cash flow statements based on accounting principles have been compiled for a long time, their application in bankruptcy modeling has been poor. The cash flow statement is a source of general financial categories for financial ratios, and an overwhelming amount of research has been limited to using only one cash flow statement category: cash flow from operating activities. However, the useful information from these statements is much broader and it could be expected that it is essential for bankruptcy prediction.

The aim of this paper is to conduct empirical research on the usefulness of cash flow components in bankruptcy modeling. However, these components are matched as cause and effect to find a new explanation for bankruptcy. The basic explanation for such a procedure is that cash flow components provide a possibility to capture the relationship between components as a sequence in time. In section 2, a piece of research is presented on the usefulness of the cash flow component in bankruptcy modeling compared to to classic cash flow ratios. In section 3, hypothesis development is considered. In section 4, a sample and the methodology are described, while in section 5 the empirical results are presented.

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1. The state of research

The aim of this section is to discuss the empirical outcomes of research presented in the literature on the possibility and usefulness of different components of cash flow as predictors of bankruptcy compared to typical accrual ratios.

A possibility to apply cash flow in financial ratios for the purpose of bankruptcy modeling was perceived quite early. The first piece of quantitative research devoted to bankruptcy was focused more on finding financial ratios which would be useful in bankruptcy prediction and thus, the ratio computation formula itself i.e. accrual ratio or cash ratio was of secondary importance. W. Beaver (1968) implemented a cash flow coverage liabilities ratio, but in fundamental research by E. Altman (1968) cash flow ratios were not used at all. Moreover, in another piece of classical research by E. Altman and others (1977), financial ratios excluded cash flow ratios. Only in the 1980s were some basic questions as to the usefulness of cash flow ratios in bankruptcy prediction raised.

Cash flow based ratios in bankruptcy modeling are quite popular but, to be honest, they are certainly not as popular as classic, „accrual” ratios. Cash flow based ratios in bankruptcy studies were used by many, for example Casey, Bartczak (1985); Gentry, Newbold, Whitford (1985); Gombola, Haskins, Ketz, Williams (1987); Aziz, Lawson (1989); Sharma (2001); Murty, Mistra (2004).

Aziz and Dar (2006) have made a broad review of bankruptcy research on crucial methodological problems such as the kind and quality of the statistical technique, the type of theoretical model framework, and the validation of research results etc. They observed that only 7% of bankruptcy models are based purely on cash flow ratios but only 33% use other information than that included in a financial statement. Their review definitely shows that the cash flow statement as a source of cash inflow and outflow components is of marginal interest to researchers.

Only some research has shown indirectly that the components of cash flow and the relationships between cash flow categories and their components can be significantly combined with bankruptcy. Dambolena and Khoury (1980) considered that the predictive capability of a model depends not only on the value of a given ratio but also on its variability in time, measured by standard variation. The observed variability of ratios in time lead to the conclusion that predictive capability is also influenced by ratio values over time, rather than only in the period preceding bankruptcy. From a more practical point of view, Foster and Ward (1997) also showed that the tendency of cash flow activities i.e. operating, investing and financing are useful for an auditor. Therefore, it can be stated that cash flows are different in the periods preceding bankruptcy or recovery.

One of the first studies to pioneer bankruptcy modeling with cash flow was the research of Gentry et al. (1985). They tested the usefulness of many components of funds flow, including funds flowing from operating activities, a change in working capital, investment expenditures, and debt coverage expenditures. All those components, including a change in cash balance, were combined in the model, and each
of the components was expressed as a financial ratio by dividing it by net funds flow. The procedure of Gentry et al. has its origin in the construction of the funds flow statement, which was the prototype of the cash flow statement.

The primary focus of Al-Attar, Hussain and Yan Zuo (2008) was to find out whether accruals are important for cash flow prediction, including bankruptcy firms. However, in different models they decomposed earnings affecting cash flow into inventory, account receivable, accounts payable, amortization and other factors. These are the components of cash flow. They also used these components deflated by assets, which actually is the inclusion of cash flow based ratios. They found that the current cash flow has an explanatory power and a strong positive association between the cash flow and the level of bankruptcy.

An important role of working capital components is still valid and it could even be stated that they are proliferating. Russell and Izzo (2009) have tested some hypotheses about improvement in accounts receivable management and inventory and accounts payable management resulting in a change in cash flow per share ratio. They proved that working capital management, and cash flow per share ratio, had changed significantly over the years.

The approach of Gentry et al. was further developed by Laitinen (1994), who tested which components of operating cash flow from a statement are the most effective in bankruptcy prediction. As in Donaldson’s model, Laitinen defined bankruptcy as a shortage of cash flow for payment of liabilities. Bankruptcy can be predicted by observing the behavior of the operating cash flow in the period preceding bankruptcy. Laitinen expressed this relationship as the ratio of liabilities coverage to the individual components of operating cash flow.

According to Laitinen’s model, the level of cash flow and every component must be different for bankrupts and non-bankrupts; for example, accounts receivable and inventories should be lower in bankruptcy-threatened firms than in non-bankrupt firms, and accounts payables, wages and so on, should be higher for bankrupts than for non-bankrupts. The differences can be small, medium or large, depending on the cash shortage.

Laitinen hypothesized that cash flow is lower for bankrupts than for non-bankrupts and that individual crucial components of the operating cash flow from a cash flow statement are different for bankrupts and non-bankrupts. Laitinen’s research revealed that the difference between the operating cash flow of bankrupts and non-bankrupts is important, it increases as a firm approaches bankruptcy, and the components of operating cash flow are almost the same for bankrupt and non-bankrupt firms.

Laitinen’s research was an important attempt to link the individual components of cash flow, as a change process over time, to corporate bankruptcy or survival. However, Laitinen’s conclusions included only cash flow components while omitting other crucial cash flow categories, and it was limited to the differences between cash flow components, without verification of a bankruptcy model.
An idea similar to the one described above was examined by Laitinen and Laitinen in their next research paper (1998). They tried to connect inventory cash management models (such as Baumol’s, Miller-Orr’s, and others’ models) and time-series behavior of cash with bankruptcy modeling. However they concluded by admitting that cash management models provide incremental information but they fail to provide additional information on traditional ratio analysis.

To generalize Laitinen and Laitinen’s opinion it can be stated that the relationship between cash and cash parameters over time is a weak indicator of bankruptcy. Obviously their opinion is against the usefulness of cash flow components in bankruptcy prediction.

Rujoub et al. (1995) used a much broader set of cash flow components than Laitinen did, including in their research not only cash flow from investing and financing activities, but also crucial components such as debt payments and dividends. The ratios were constructed as the relationship between cash flow outflows in individual categories and possible inflow sources. For example, the long-term liabilities payment ratio was the relationship of paid to taken debts, or the relationship of cash outflow to cash inflow. The research of Rujoub et al. showed that ratios based on cash flow components (cash ratios) have a higher predictive capability in the years before bankruptcy than do traditional (accrual) ratios.

The Rujoub-Cook-Hay model suggests that, in bankruptcy prediction, an important role is played not only by components of operating cash flow, but also by other sources of cash inflow and outflow. The construction of ratios, in which some categories from cash flow statements were logically linked, is also crucial. However, the model was non-unified in some ratios, as the same ratios were sometimes outflows and sometimes sources of cash. The model also lacked logical connections between ratios, leaving them as a loose set of ratios, as in a typical bankruptcy model. Finally, the model omitted links between ratios in time. Still, Rujoub et al. confirmed the significance of cash flow components.

Grover (2003) used the concept of a cash reservoir of inflow and outflows in non-bankruptcy prediction. The model included components of cash flow statements in the form of ratios (i.e. cash flow components deflated by assets). He found out that the cash flow based model is superior in bankruptcy prediction compared to Altman’s model, which is in turn a useful model in selection of non-bankrupts. Although Grover’s research constitutes substantial evidence of the usefulness of cash flow components, it is still a typical static approach i.e. cash flow measures are calculated at a definite moment in time, not over a period or periods.

Much more precise research showing some cash flow properties of bankruptcy modeling was that of Maślanka (2008). On a sample of 40 bankrupts and 40 non-bankrupts listed on the Warsaw Stock Exchange in 2000–2004, Maślanka observed the signs of cash flows (e.g. a positive sign of operating cash flow) in a group of bankrupt and non-bankrupt firms a year, two years and three years before bankruptcy. Maślanka concluded that cumulative net cash flow and cumulative operating
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cash flow are negative for bankrupts and positive for non-bankrupts, and that cumula-
tive investing cash flow is negative for both groups, but financing of cash flow
is negative only for non-bankrupts.

Maślanka (2008) also made some important observations about the links be-
tween some special cash flow components – the signs of cash flow for operating,
investing and financing (for example +,+,-) in a group of bankrupt and non-
bankrupt firms: In 80% of the bankrupt group, the sign patterns include positive
operating cash flow, while the sign was positive for only 55% of the non-bankrupt
group; and the number of firms characterized by given sign patterns in a group of
bankrupt and non-bankrupt firms does not show any clear discrimination of sign
patterns between the groups.

In the two years before bankruptcy, Maślanka’s research showed that positive
operating cash flow characterized 72% of non-bankrupt and 60% of bankrupt
firms, so the distribution was almost proportional.

Maślanka’s research is valuable because it provides signals that sign patterns of
cash flow are useful in bankruptcy prediction, and it shows that patterns of cash
flow can be bankruptcy predictors as much as two years before bankruptcy. How-
ever, Maślanka’s research contains no bankruptcy model incorporating sign pa-
tterns of cash flow as predictors and omits the influences between signs, although it
is logical that they have to change sequentially (for example, positive financing
cash flow will have to become negative because of debt payments). Maślanka’s
research also focuses only on signs of cash flow, ignoring the amount of cash flow.
The conclusions for the present research are as follows:

- The literature mainly pays attention to classic cash flow ratios used as one or
two ratios in a set of well-known accrual ratios, ignoring plenty of information
from cash flow components included in a cash flow statement.

- Only some researchers tested the component of cash flow as a relation between
the components, concluding that they are important and can be used in bank-
ruptcy modeling. It seems that only Maślanka used the broad definition of cash
flow components considering signs of cash flow characteristics. However, his
research lacked formal bankruptcy model testing, and the final conclusion of the
usefulness of cash flow components (including cash flow signs) was not stated.

The aim of this paper is therefore a more general description and empirical test-
ing of cash flow components in a bankruptcy model structure.

2. Hypotheses development

The problem of using cash flow statement components in bankruptcy modeling
omitted in all the referred literature is that the relationship between them is not as
simple as described in the literature, and strictly speaking they are quite compi-
lcated. Table 1 shows the relationship between operating, investing and financing
cash flows (and in general components of each of them) in a given single period, and the relationship between two subsequent periods. To simplify the relations only these two periods are considered.

**Table 1. Influence of cash flow type on each other in a single period and between two subsequent periods**

<table>
<thead>
<tr>
<th>Previous period</th>
<th>Next period</th>
<th>Operating cash flow</th>
<th>Investing cash flow</th>
<th>Financing cash flow</th>
<th>Net cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cash flow</td>
<td>1</td>
<td>X/→</td>
<td>X/→</td>
<td>X/→</td>
<td>X/→</td>
</tr>
<tr>
<td>Investing cash flow</td>
<td>1</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
</tr>
<tr>
<td>Financing cash flow</td>
<td>1</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
</tr>
<tr>
<td>Net cash flow</td>
<td>1</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
<td>→/→</td>
</tr>
</tbody>
</table>

Where:
1  – positive sign or amount of cash flow,
0  – non-negative sign or amount of cash flow
X  – lack of relation between cash flow components in the present period,
→  – arrows show a cause-effect relationship between a given component of cash flow in the previous period / from the previous to the next period.

The influence of one kind of cash flow (for example, operating cash flow) on another kind of cash flow (for example, investing cash flow) can be in one of two ways (table 1):

- in the previous period, the cash flow of a given kind influences other kinds of cash flow. For example, a surplus of operating cash flow can be spent on debt payment, thus affecting financing cash flow;
- in the prevailing period (i.e. the period before bankruptcy), cash flow can influence each kind of cash flow in the next period. For example, financing cash flow in the former period is also negative in the next period because debt-paying is continued.

In the first case, cash flow of a given kind in the former period can influence other cash flows in that period, but it cannot influence itself (X excludes such examples in table 1). In addition, it is theoretically possible to separate the influence of one kind of cash flow on another kind; for example, the influence of operating cash flow on investing cash flow, and the influence of investing cash flow on operating cash flow can be separated, but the construction of the cash flow statement makes it impossible to set the direction of this influence, that is, whether operating cash flow influences investing cash flow or vice versa. In table 1 this relation is presented as arrow from cash flow as the cause in previous period to the effect also in previous period.
In the second case, all possible combinations between cash flows are probable (therefore, the operating cash flow from a previous period can influence the operating cash flow in next period). Net cash flow is also included because it indirectly shows the amount of cash expenditures (i.e. cash spent from the beginning stock) that are stocked in the present period (described in table 1 as an arrow from a given component of cash flow from subsequent period to a given component of cash flow in the next period). An influence of net cash flow from the former period on net cash flow of the next period is examined but, theoretically, it could be unimportant because cash flow in the next period will be dependent on cash flow in this period (i.e. before bankruptcy). The cash flow of a given type from a former period can change all types of cash flow in the next period, including cash flow of the same kind. For example, negative cash flow resulting from investment expenditures in the former period can influence investing cash flow in the next period because, after the investment process, old machinery is usually sold out. In table 1 this relation is presented as arrow from cash flow as the cause in previous period to the effect in next period.

From the above, it is therefore clear that it is necessary to model in bankruptcy prediction all these relation i.e. relationships between components of cash flow in a single period (previous) and influence on components of cash flow between periods i.e. from previous on the next (subsequent) period. Testing methodology has to capture both relations and should also provide classic testing methodology of bankruptcy model effectiveness as a prediction model. The conclusion is that the scientific task is complicated.

From a literature review, components of cash flow can be divided as follows:

- type of cash flow that are operating, investing and financing activities;
- items of all of these types e.g. working capital change in operating cash flow;
- signs of cash flow which are in each type positive or non-negative.

To simplify the present research, only the type of cash flow (as a good representation of overall changes on items) and the sign of cash flow (as a "non-classic" cash flow component) are considered. From table 1 it can be concluded that several properties of cash flow can be useful in bankruptcy prediction:

1. The signs of cash flow of a given kind in one period can influence the signs of cash flow in the next period preceding bankruptcy. A shortage of cash in the former period forces actions to be taken in the next period if present actions are insufficient to diminish the shortage. Therefore, one bankruptcy predictor is a sequence of transitions from given sign patterns of cash flow in the former period to sign patterns of cash flow in the next period.

2. Signs and amounts of cash flow in a former period can force signs and amounts of cash flow in the next period preceding bankruptcy. The argumentation here is the same as before but includes not only the sign patterns of cash flow but also the amount of cash flow. Therefore, a sequence of signs and amounts in the former period to signs and amounts in the next period is crucial.
3. Because cash ratios that characterize the cash flow of a given kind in a former period can influence these ratios in the next period, a sequence of ratios in the former period to ratios in the next period can be considered. Cash ratios are ratios that are constructed mainly from components of cash flow from the cash flow statement.

Because cash flow analysis in bankruptcy modeling can be seen as competing with classic accrual based ratios modeling, the usefulness of a cash flow component based analysis should be effective in the classic accrual based approach. Cash flow is significant in predicting bankruptcy because:

- the cash flow of given signs in a previous period affects the signs of cash flow in a subsequent period. For example, positive financial cash flow – meaning incurred debts – implies negative cash flows in the next period, when debt is settled. This kind of sequence is described in hypothesis H1;

- the cash flow of given sign and amount in a previous period influences the signs and the amounts of cash flow in the next period. For example, a value of positive operating cash flow enables a firm to make capital expenditures in the next period, resulting in a negative investing cash flow. This kind of sequence is described in hypothesis H2.

These proprieties of cash flow are taken into account in the research hypotheses. In the matter of sign pattern sequences of cash flow, hypothesis H1 is as follows:

H1. The sign pattern sequence of a given kind of cash flow in a one-year period before bankruptcy that results from the sign pattern of a given kind of cash flow in a two-year period is significant for bankruptcy prediction, and the total predictive capacity of the model built on this sequence is not less than the predictive capacity of the model based on accrual ratios for the same period.

In the matter of signs and amounts significance, hypothesis H2 is as follows:

H2. The sequence of sign patterns and amounts of a given kind of cash flow in a one-year period before bankruptcy that results from sign patterns and amounts of a given kind of cash flow in a two-year period is significant for bankruptcy prediction, and the total predictive capacity of the model built on this sequence is not less than the predictive capacity of the model based on accrual ratios (accrual model) for the same period.

The reasoning for hypotheses H1 and H2, as well as the next, H3, derives from the construction of a cash flow statement that can be incorporated into the bankruptcy model. Therefore, the cash flow statement incorporates a complete list and sum of cash inflow and cash outflow components into specific types, and presents the amount of cash stockpiled at the beginning of a previous period and at the beginning of the period preceding bankruptcy. Comparison to accrual model is necessary to check whether sequence of cash flow gives any additional bulk of information. From research of cash flow usefulness it is known that cash flow is useful. However the problem is whether it could be the only one source of information or additional (major or minor).
The cause-effect links between components of cash flow, such as sign patterns and amounts, can be modeled traditionally, that is, as relative measures that are financial ratios. Hypothesis H3 formalizes this:

H3. The cause-effect sequence of financial ratio values characterizing cash flow of a given kind in a one-year period, and the values of those ratios from previous year (two-year period before bankruptcy), are significant for bankruptcy prediction, and the total predictive capacity of the model built on this sequence is not less than the predictive capacity of the model based on accrual ratios (accrual model) for the same period.

The reasoning for hypothesis H3 derives from the limitations of the cash flow statement itself, because this statement does not provide some important information that could be useful in bankruptcy modeling. For example, it does not incorporate logical links between individual sources of cash and its use. In other words, neither the precise outflow of a given amount of cash, nor in what way this shortage is covered, is known. Another point of missing information is that cash stock does not reflect the full range of possibilities for spending this stock in approaching periods. Because the purpose of a corporation is not to collect money, an ending balance of cash is not an outcome of corporate activity because firms can get cash from current operations.

A final example of missing information relates to the fact that cash from the previous period is currently increased or decreased through operating activities, of which the only result is income. Therefore, the reasons for a change in working capital for investment decisions and financing decisions are lacking. The fundamental impulse lies in sales dynamics in the present period or, alternatively, the sales forecast anticipated by activities incorporated into the cash flow statement.

To overcome these limitations while preserving the positive attributes of the cash flow statement, the relationship among the components of cash flow can be modeled with financial ratios. However, to differentiate these ratios from typical ratios based on cash flow, it is necessary that these ratios fulfill two conditions: they must incorporate at least the individual kinds of cash flow and they must characterize the links between types of cash flow.

3. Research procedure

In order to verify the hypotheses, the research was carried out on a Polish corporate population other than banks and insurance companies. Because it was necessary to use cash flow components, the population was limited only to corporations that have a legal obligation to draw up a cash flow statement. This obligation in Poland is limited to joint-stock companies. The definition of bankruptcy used was the legal definition based on the Polish Bankruptcy Act.

The analytical sample contained 38 companies that went bankrupt in 2004. Because hypotheses testing considered only the two years before bankruptcy, financial
statements were collected from years 2001–2002. The bankruptcy sample considered 24 industries by PKD (Polish Industry Code). The sample included all bankrupt companies, which was 38 companies. These companies were compared to 38 non-bankrupt companies sampled from the same industries as the bankrupts in 2002 for the same years. Companies from the non-bankrupts which finally did go bankrupt in 2004 were also eliminated. Therefore, the total number of companies in the analytical sample was 76, after the previous non-bankrupt selection.

The predictive capacity of the models estimated for 2001–2002 was verified on a validation sample for 2002–2003. The validation sample contained 33 companies that went bankrupt in 2005 and they were mirror companies of that drawn to analytical sample on industry code. The total number of companies was 66. Because of the data gap in this period in the analytical sample, non-bankrupt companies which finally went bankrupt in the following year were included.

For hypotheses testing, logit regression was used because it gives a clear-cut probability estimation of bankruptcy, and it provides an easy way to incorporate signs of cash flow (as 0 and 1 codes) and of course numerical values (as values of cash flow and as values of financial ratios). It is also classical bankruptcy modeling tool in easy way giving ability to incorporate a cause-effect relation.

The likelihood ratio test was used to test the estimated models at statistic D (Kleinbaum et al., 1998; Stanisz, 2006, vol. I). Collinear variables were not eliminated because other research carried out by the author on the Polish economy (2005a, 2005b) concluded that elimination of collinear variables worsens the classification and predictive capacity of models. These previous studies showed that reducing the number of variables and eliminating outliers improve the models’ classification and predictive power.

In order to identify explanatory variables that are insignificant for classification capacity, forward stepwise regression and auxiliary correlation analysis were used. A choice of variables for the reduced model (i.e. received on the basis of forward stepwise regression) depends on the following criteria:

- the classification capacity of a model has to be higher with an additional variable than a model without this variable,
- the ex post probability for the model with an additional variable has to be comparable to or lower than that for the model without this variable,
- a goodness of fit measure for the empirical data, which is Nagelkerke’s pseudo $R^2$, must be better for the model with the additional variable than for the model without this variable.

Estimating the model’s significance with an additional variable, compared to a simpler model, typically uses an incremental likelihood ratio test and statistics $\Delta D$ (Tabachnick, Fidell, 2007; Stanisz, 2007, vol. II). The assessment of goodness of fit to the empirical data used the pseudo $R^2$ of Nagelkerke, which is set in the range [0, 1] and includes the size of the sample. Classification criterion is standard for a logit model, that is, bankruptcy is for values not less than 0.5; for values lower than 0.5, the firm is a going concern.
On account of many possibilities of econometric techniques and some ways of hypotheses testing it was decided to use several different models including slightly different econometric techniques and ways of hypotheses testing. Such a procedure makes some information noise but it excludes a priori choice of model.

Because elimination of outliers improves the effectiveness of the bankruptcy model, outliers were eliminated on the basis of empirical residuals. A „mechanical” procedure was implemented (called „method 1” in the paper) in which firms were eliminated from the analytical sample if the residual of the given logit model was in the range \([-1; 0.75]\) or \((0.75; 1]\). This procedure was successfully applied (and more broadly described) in previous researches of author (Wędzki, 2005a, 2005b).

Another method for eliminating outliers was used, here called „method 2”, because classical analysis of empirical residuals does not always eliminate all outliers. In this method, deciles are calculated for individual explanatory variables separately for bankrupts and non-bankrupts. Next, marginal deciles are compared (i.e. for non-bankrupts, the first decile or the lowest cash flow of a given kind, and for bankrupts the ninth decile or the highest cash flow of a given kind). Overlapping the first deciles of non-bankrupts and the ninth decile of bankrupts provides the field in which bankrupts have extraordinarily high cash flows and non-bankrupts have extraordinarily low cash flows. Companies that belong in this field are considered outliers and are eliminated. Also, to preserve the characteristics of the matched sample, a pair of companies is eliminated when at least one of the pair is classified as outlier. Because each explanatory variable can characterize different specific outliers, only the cases that are classified as outliers in more than one independent variable are eliminated. In order to avoid too many values in the logit model, they are expressed in the millions of currency units (PLN), but their logarithms are not found because of possible negative cash flows.

As to the model, possible differences in the econometric methodology of hypothesis H1 and others are tested with slightly differently constructed models. For hypothesis H1, model M1 is estimated on the full sample containing all companies (cases), and model M2 on the sample without outliers (eliminated method 1) and with a reduced number of variables. Method 2 cannot be applied because the variables are binary.

For hypothesis H2, model M3 is estimated on a full sample in which the explanatory variables from the previous and following period (i.e. 8 explanatory variables) are a set of variables. It is therefore a model including „inexplicit” tendency. A similarly built model is model M4, but it is estimated on the sample without outliers and has a reduced number of variables. Cash flows of a given kind from the former and the next period are eliminated. The idea is to completely eliminate any given kind of cash flow because it is possible that a positive or negative cash flow of a given kind from only one period will remain in the model (e.g. only from the second year before bankruptcy). The methodology of the logit model is the same as in H1, where outliers are eliminated by observing the residuals of method 1 (M4) or method 2 (i.e. outliers are in the „gray” area marked by marginal deciles) (M4).
In hypothesis H3, the cash flow of given kind and the relationship between cash flows can also be modeled with financial ratios. However, in order to characterize the properties of the ratio analysis while not duplicating the classical approach to ratio analysis on cash flows, some additional conditions must be satisfied: only the components of cash flow are used, all kinds of cash flow are included, and a character for the relationship between the components of cash flow is incorporated.

An ideal solution from the theory of finance point of view would be a set of ratios inferred from corporate aims and described by an economic-value-added model, a residual income model or something similar. The fundamental problem was to construct ratios that could capture the cash flow sequence and residual income model assumptions, as well as the cause-effect relationship (i.e. as a mathematical model). The set of ratios worked out concentrates mainly on the relationship between types of cash flow. It is not necessary to incorporate all individual components of cash flow (e.g. cash outflows) because the possible number of combinations of cash inflows and outflows is high and difficult to model effectively. It is sufficient to incorporate only total surpluses or deficits of a given kind of cash flow because they precisely describe company decisions connected with cash flows in any period under consideration.

Two set of ratios were considered:

1. Set 1 (called „way 1”) in which the cause-effect relationships are separately modeled at two years and one year before bankruptcy. Therefore, cash flows between types of cash flow (e.g. operating and investing) are included in a given period, so cash flows within the period are more important.
2. Set 2 (called „way 2”) in which cause-effect relationships between cash flows are modeled at two years and one year before bankruptcy, so cash flows between periods are more important.

In the current research, the Set 1 ratios examined are:

- sales dynamics (DS) = net sales in the next period / net sales in the former period;
- cash sales profitability (GS) = cash flow from operating activities / net sales;
- degree of surplus investing cash (PI) = (cash flow from operating activities + cash flow from investing activities) / cash flow from operating activities;
- degree of surplus financing cash (PF) = (cash flow from operating activities + cash flow from investing activities + cash flow from financing activities) / (cash flow from operating activities + cash flow from investing activities);
- level of cash sources (RG) = net cash flow / cash and cash equivalents at the beginning of the period.

The Set 2 ratios are based on dynamics of sales (DS) as a major factor and dynamics of the individual type of cash flow (cash flow from the present period divided by cash flow from the previous period) i.e.: dynamics of cash flow from operating activities (D_{CFO}), dynamics of cash flow from investing activities (D_{CFI}), dynamics of cash flow from financing activities (D_{CFF}) and dynamics of net cash flow (D_{NCF}).
Hypothesis H3 is based on the relationship of the ratios one year before bankruptcy and two years before bankruptcy. Consequently, the significance and usefulness of ratios in predicting bankruptcy was verified with a logit model. Model M5 was estimated such that model M5\(^1\) is the model on the full sample based on ratios computed by way 1, and model M5\(^2\) is the model for which ratios were computed with way 2. Model M6 is estimated on the reduced sample from which outliers were eliminated with method 1, and model M6\(^*\) is the model in which outliers were eliminated with method 2. For both samples, ratios were calculated with way 1 or 2, so the following models were tested:

- model M6\(^1\) on the reduced sample by method 1 and on ratios by way 1,
- model M6\(^2\) on the reduced sample by method 1 and on ratios by way 2,
- model M6\(^*1\) on the reduced sample by method 2 and on ratios by way 1,
- model M6\(^*2\) on the reduced sample by method 2 and on ratios by way 2.

The second part of the test for hypotheses H1, H2 and H3 relies on comparing the classification capacity of estimated models in order to find the most effective models on which to verify the validation sample. These models will be compared to the accrual models (using such ratios as current ratio, coverage ratio, ROS, ROE and others). By analogy, as previously, collinearity of variables in estimated accrual models is possible because, to provide comparability of accrual and sequence of cash flow models, a „hidden” tendency is incorporated into the model. Thus, accrual ratios of the same kind are included in the model, period by period (i.e. ratios from 2001 and 2002).

The following procedure of forward logit regression is applied: a pair of ratios of a given kind is included in the model from 2001 and 2002 (e.g. ROE from 2001 and 2002). Therefore, from one pair of ratios, the next pair is added sequentially. The purpose is to avoid a situation in which only the model’s one-year ratio is significant, such as from 2002 or 2001. According to this procedure, the given ratio is incorporated into the model only under the condition that it is significant for two years. Therefore, this model, with a „hidden” tendency, is comparable with models M1–M6.

For hypothesis H1, models M1 and M2 were estimated (Z means sign of operating CFO, investing CFI, financing FCF, and net NCF cash flow, 02, 01 are number are 2 and 1 year before bankruptcy). Their parameters are in table 2. Table 3 presents the \(\chi^2\) statistics of those models, the pseudo R\(^2\), the testing probability p, the % of correctly classified bankrupts and non-bankrupts, and total classification capacity. Model M1, estimated on the full sample, is statistically significant and well fitted, and its overall efficiency is 77.63% (table 3).
Table 2. Parameters of logit models

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter of models</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>2.417 – 1.631 Z02_CFO − 0.969 Z01_CFO + 1.338 Z02_CFI + 0.469 Z01_CFI − 0.536 Z02_CFF − 0.184 Z01_CFF − 0.977 Z02_NCF − 0.817 Z01_NCF</td>
</tr>
<tr>
<td>M2</td>
<td>8.234 – 4.425 Z02_CFO − 3.085 Z01_CFO + 3.164 Z02_CFI + 2.716 Z01_CFI − 1.541 Z02_CFF − 1.493 Z01_CFF − 3.974 Z02_NCF − 2.515 Z01_NCF</td>
</tr>
<tr>
<td>M3</td>
<td>0.067 – 0.329 K02_CFO + 0.682 K01_CFO − 0.169 K02_CFI + 0.634 K01_CFI − 0.390 K02_CFF + 0.674 K01_CFF + 0.098 K02_NCF − 0.794 K01_NCF</td>
</tr>
<tr>
<td>M4</td>
<td>0.086 – 0.278 K02_CFO + 1.104 K01_CFO + 0.569 K02_CFI + 1.168 K01_CFI − 0.03 K02_CFF + 0.749 K01_CFF − 2.374 K02_NCF − 2.126 K01_NCF</td>
</tr>
<tr>
<td>M4′</td>
<td>0.016 – 1.320 K02_CFO + 10.21 K01_CFO − 0.095 K02_CFI + 10.75 K01_CFI − 1.607 K02_CFF + 9.86 K01_CFF − 2.543 K02_NCF − 11.038 K01_NCF</td>
</tr>
<tr>
<td>M5</td>
<td>2.283 + 0.323 DS02 − 2.560 DS01 + 0.028 D02_CFO + 9.742 D01_CFO + 0.019 D02_CFI + 1.034 D01_CFI − 0.282 D02_CFF − 0.026 D01_CFF − 0.278 D02_NCF − 0.047 D01_NCF</td>
</tr>
<tr>
<td>M6</td>
<td>6.133 − 0.245 DS02 − 6.260 DS01 + 0.055 D02_CFO + 0.163 D01_CFO + 0.076 D02_CFI + 0.237 D01_CFI − 0.497 D02_CFF − 0.160 D01_CFF − 0.497 D02_NCF − 0.092 D01_NCF</td>
</tr>
<tr>
<td>M6′</td>
<td>3.390 − 0.892 DS02 − 2.621 GS02 − 1.900 GS01 + 0.099 PF02 + 0.105 PF01</td>
</tr>
<tr>
<td>M6′′</td>
<td>3.068 − 1.203 DS02 − 1.625 DS01 − 0.087 D02_CFO + 0.068 D01_CFO − 0.658 D02_CFF − 0.037 D01_CFF</td>
</tr>
<tr>
<td>M_{02-01}</td>
<td>1.697 + 3.846 PZB_{02} − 1.782 PZB_{01−} + 0.238 POD_{02} − 1.373 POD_{01} + 0.258 ZI_{02} − 1.485 ZI_{01} + 3.210 RNS_{02} − 1.952 RNS_{01−} + 3.579 ROA_{02} + 3.311 ROA_{01} + 0.075 ROE_{02} − 0.562 ROE_{01}</td>
</tr>
<tr>
<td>\bar{M}^R_{02-01}</td>
<td>2.727 + 2.571 PZB_{02} − 5.215 PZB_{01−} + 0.152 POD_{02} − 2.547 POD_{01} − 11.223 RNS_{02} + 0.686 RNS_{01−} − 7.207 ROA_{02} − 4.413 ROA_{01}</td>
</tr>
</tbody>
</table>

Source: own research.

Table 3. Characteristics of model estimated on analytical sample

<table>
<thead>
<tr>
<th>Model</th>
<th>D</th>
<th>p</th>
<th>R²</th>
<th>Bankrupt as bankrupt (in %)</th>
<th>Non-bankrupt as bankrupt (in %)</th>
<th>Total classification capacity (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>24.757</td>
<td>0.00171</td>
<td>0.511</td>
<td>71.05</td>
<td>84.21</td>
<td>77.63</td>
</tr>
<tr>
<td>M2</td>
<td>52.336</td>
<td>0.00000</td>
<td>0.848</td>
<td>81.81</td>
<td>84.84</td>
<td>83.33</td>
</tr>
<tr>
<td>M3</td>
<td>17.313</td>
<td>0.02704</td>
<td>0.390</td>
<td>81.57</td>
<td>65.78</td>
<td>73.68</td>
</tr>
<tr>
<td>M4</td>
<td>52.024</td>
<td>0.00000</td>
<td>0.814</td>
<td>86.11</td>
<td>75.00</td>
<td>80.56</td>
</tr>
<tr>
<td>M4′</td>
<td>51.154</td>
<td>0.00000</td>
<td>0.861</td>
<td>77.419</td>
<td>87.09</td>
<td>82.26</td>
</tr>
<tr>
<td>M5</td>
<td>25.624</td>
<td>0.00429</td>
<td>0.522</td>
<td>73.68</td>
<td>71.05</td>
<td>72.37</td>
</tr>
<tr>
<td>M5′</td>
<td>25.222</td>
<td>0.00001</td>
<td>0.759</td>
<td>78.78</td>
<td>81.81</td>
<td>80.30</td>
</tr>
<tr>
<td>M6</td>
<td>47.975</td>
<td>0.02332</td>
<td>0.391</td>
<td>64.86</td>
<td>64.86</td>
<td>64.86</td>
</tr>
<tr>
<td>M6′</td>
<td>39.223</td>
<td>0.01476</td>
<td>0.396</td>
<td>76.47</td>
<td>76.47</td>
<td>76.47</td>
</tr>
<tr>
<td>M_{02-01}</td>
<td>39.639</td>
<td>0.00008</td>
<td>0.69</td>
<td>73.7</td>
<td>76.3</td>
<td>75.0</td>
</tr>
<tr>
<td>\bar{M}^R_{02-01}</td>
<td>48.762</td>
<td>0.00000</td>
<td>0.812</td>
<td>85.3</td>
<td>88.2</td>
<td>86.8</td>
</tr>
</tbody>
</table>

Source: own research.
For verification of hypothesis H2, model M3 (table 2) is estimated on the full sample and its characteristics are shown in table 3. It is clear that this model is statistically significant but only moderately fitted to the data. Model M4 on the reduced sample was created from observations of the M3 model residuals (method 1). Table 3 shows that this model is statistically significant, very well fitted to the data (pseudo $R^2$ is 81.4%), and effective at a level of 80.56%.

Model M4 is estimated on the reduced sample using the alternative method 2 – the method of marginal deciles. Therefore, the 9th decile of non-bankrupts and the 1st decile of bankrupts were set for individual ratios computed by way 1. Bankrupts and non-bankrupts were eliminated from the sample when at least two ratios were positioned in the gray (critical) area. The reduced sample by method 2 was also set on the sample for which ratios were calculated by way 2. The parameters of model M4 are shown in tables 2 and 3. In table 2 K means sing and amount of given cash flow, years described as for models M1 and M2.

Models M5 and M6 were estimated in some variations. Model M5$^1$, which is the model on ratios computed with way 1, failed to estimate any iteration method. In table 2 DS is sales dynamics and D is dynamics for CFO, CFI, CFF and NCF, cash sales profitability is GS, degree of surplus financing cash is PF. The parameters of the model based on ratios of way 2, model M5$^2$, are in table 2 and the model’s characteristics are in table 3. From table 3, it is clear that the model is statistically significant and moderately fitted to the data. The classification capacity is 72%.

Because the estimation of model M5$^1$ did not succeed, it was impossible to set the reduced sample by method 1 – that is, on the analysis of residuals – so model M6$^1$ also could not be estimated. Model M6$^2$ was estimated on the reduced sample based on the analysis of residuals for financial ratios and on the elimination of ratios with way 2. The characteristics and parameters of model M6$^1$ are shown in tables 2 and 3. Model M6$^1$ is statistically significant but moderately fitted to the data, and it has a low classification capacity. For almost half of the cases for which independent variables were eliminated, iteration convergence failed and, as a result, the model could not be estimated, so the high volatility of ratios from the logit model cannot be estimated either. The model based on ratios computed with way 2, model M6$^2$, is presented in tables 2 and 3. This model has high classification capacity, low empirical probability p and high goodness of fit to the data.

The accrual model is estimated on the full sample that is, model $M_{02-01}$ for 2001 and 2002. The second model is the model on the reduced sample, that is, the sample without outliers by method 1, and the model has a reduced number of variables. It is also modeled with „hidden” tendency in model $\hat{M}_{02-01}^K$. In table 2 current ratio is PZB, coverage ratio is POD, investment financing ratio is ZI, return on sales is RNS, return on assets is ROA, return on equity is ROE. These ratios are only the ratios that have their cash flow comparisons i.e. cash flow based ratios. Some ratios as debt ratio cannot be used in this manner (Wędzki 2005a, 2005b). The aim is to compare cash flow components to comparable accrual model in the construction of ratios and econometric methodology used to select ratios.
4. Research conclusions

Models presented in table 2 were tested on a validation sample. The percentage of partial and total predictive capacity is shown in table 4.

Table 4. Partial and total predictive capacity of models estimated on validation sample

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>Name of model</th>
<th>Bankrupt as bankrupt (in %)</th>
<th>Non-bankrupt as non-bankrupt (in %)</th>
<th>Total correct predictive capacity (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>M1</td>
<td>47.8</td>
<td>73.90</td>
<td>63.0</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>56.5</td>
<td>69.60</td>
<td>63.0</td>
</tr>
<tr>
<td>H2</td>
<td>M3</td>
<td>78.3</td>
<td>52.20</td>
<td>65.2</td>
</tr>
<tr>
<td></td>
<td>M4</td>
<td>60.9</td>
<td>52.20</td>
<td>56.5</td>
</tr>
<tr>
<td></td>
<td>M4'</td>
<td>60.9</td>
<td>69.60</td>
<td>65.2</td>
</tr>
<tr>
<td>H3</td>
<td>M5'</td>
<td>60.90</td>
<td>34.80</td>
<td>47.80</td>
</tr>
<tr>
<td></td>
<td>M6'</td>
<td>43.50</td>
<td>30.40</td>
<td>37.00</td>
</tr>
<tr>
<td></td>
<td>M6'1</td>
<td>39.10</td>
<td>56.50</td>
<td>47.80</td>
</tr>
<tr>
<td></td>
<td>M6'2</td>
<td>87.0</td>
<td>8.70</td>
<td>47.80</td>
</tr>
<tr>
<td>H1–H3</td>
<td>M_{02-01}</td>
<td>51.50</td>
<td>52.20</td>
<td>63.0</td>
</tr>
<tr>
<td></td>
<td>M_{R}^{02-01}</td>
<td>52.20</td>
<td>78.30</td>
<td>65.20</td>
</tr>
</tbody>
</table>

Source: own research.

Hypothesis H1 proposed that the predictive capacity of the model based on the sequence of cash flow signs of a given kind was better than the predictive capacity of an accrual model, that is, a model based on accrual ratios. The predictive capacity of a cash model based on sequence of signs (model M1) is 63%, compared to the predictive capacity of the two-year accrual model M_{02-01}, whose effectiveness is 63%. The cash model with a reduced number of independent variables on the sample and without the outliers eliminated with method 1 is model M2, whose predictive capacity is 63%. This model is compared to the two-year accrual model estimated on the same sample (reduced) and with a reduced number of variables (the same method as previous). This is model $M_{02-01}^R$ with an effectiveness of 65.20%.

These comparisons lead to the conclusion that hypothesis H1 should be accepted partially because the predictive capacity of the cash model is comparable to the effectiveness of the accrual model only if the sample is full and only if any independent variable is eliminated. Hypothesis H1, which proposed that an accrual model is more effective than a cash model, should be rejected if the sample is reduced and outliers are eliminated with the analysis of residuals.

Hypothesis H2 proposed that the predictive capacity of a cash model based on the amounts and signs of a given kind of cash flow is not lower than the predictive
capacity of a comparable accrual model. The usefulness of the amounts and signs sequence was tested by comparing the predictive power of Model M3, which is 65.2%, with the predictive power of the two-year accrual model M_{02-01}, which was 63%. In other words, the predictive capacity of the model was tested with a reduced number of variables on a reduced sample where outliers were eliminated by method 1 (M4) and method 2 (M4'). Their predictive powers were 56.5% and 65.20%, respectively. A suitable model for comparison is the two-year accrual model with a reduced number of variables and on a reduced sample by method 1 (\( \overline{M}_{02-01}^R \)). The predictive power of this model is 65.20%.

From the above comparisons it is clear that amounts and signs of cash flow give better prediction only if there is no variable reduction and outliers elimination. Method 2 of outliers elimination provides comparable results. Finally, hypothesis H2 is proved only under the above conditions. Therefore the structure of the conclusion is quite similar to that for hypothesis H1.

Model M5 and Model M6\(^1\) are the models based on ratios calculated with way 1 with a reduced number of variables and on the reduced sample by method 2. Its predictive capacity is 47.80%. The model on ratios calculated with way 2 is model M6\(^2\), which has a predictive capacity also equal to 47.80%. For both models, the comparable accrual model is model \( \overline{M}_{02-01}^R \).

The comparison of the predictive power of the cash and accrual models provides an unambiguous conclusion that hypothesis H3 must be rejected. The calculation of ratios with way 1 or way 2, and the simultaneous inclusion of the „inexplicit” tendency of these ratios, do not provide a better prediction than traditional accrual ratios.

**Concluding remarks**

On the basis of the formulated hypotheses, some general observations can be formulated according to the usefulness of the sequence of cash flow components in bankruptcy modeling that is the sequence of cash flow signs, the sequence of cash flow signs and amounts, and also inclusion of ratios in the cash model that characterize the type of cash flows. The final conclusion of the research is that the signs of cash flow and the signs and the amounts are useful in bankruptcy prediction only under some conditions. Cash flow based ratios characterizing the structure of cash flow statements are poor bankruptcy estimators.

On the basis of hypotheses H1 – H2, some general observations can be formulated according to the usefulness of the sequence of cash flow components in bankruptcy modeling:

- the predictive capacity of cash models based on the sequence of cash flow signs and the sequence of cash flow amounts and signs is comparable only to the pre-
dictive capacity of traditional accrual models and comparable only under the condition that outliers are not eliminated and variables are not reduced;

- the inclusion of ratios in the cash model that characterize the type of cash flows does not provide better predictive power, irrespective of whether ratio computation way 1 or way 2 is used.

Therefore, the sequence of cash flow components does not provide significant information for bankruptcy modeling compared to the traditional accrual ratios. However, further research can be carried out on the usefulness of the component of cash flow in mixed models where both accrual and cash ratios are used. Another research direction is the verification of the developed hypotheses on a greater number of subsequent periods. It is mainly the problem of good bankruptcy data which are still lacking in the Polish economy.

References
The sequence of cash flow in bankruptcy prediction: evidence from Poland


Summary
Although cash flow statements based on accounting principles have been compiled for a long time, their application in bankruptcy modeling has been poor. The aim of this paper is to conduct empirical research on the usefulness of cash flow components matched as cause and effect in subsequent periods (sequentially) in bankruptcy modeling. On the basis of the formulated hypotheses, some general observations can be formulated according to the usefulness of the sequence of cash flow components in bankruptcy modeling that is the sequence of cash flow signs, the sequence of cash flow signs and amounts, and also the inclusion of ratios in a cash model that characterize the type of cash flows. The final conclusion of the research is that the signs of cash flow and the signs and the amounts are useful in bankruptcy prediction only under some conditions. Cash flow based ratios characterizing structure of cash flow statements are poor bankruptcy estimators.

Keywords: bankruptcy prediction, cash flow analysis, bankruptcy modeling, cash flow modeling.

Streszczenie
Sekwencja przepływów pieniężnych w prognozowaniu bankructwa
Mimo że rachunek przepływów pieniężnych jest sporządzany zgodnie z regulacjami rachunkowości od dłuższego czasu, to jednak jego wykorzystanie w prognozowaniu bankructwa przedsiębiorstw jest ubogie. Celem artykułu jest przedstawienie wyników badań nad użytecznością składników rachunku przepływów pieniężnych ujętych w relacje przyczynowo-skutkowe w następujących po sobie okresach (sekwencjach) w modelowaniu upadłości. Testowane hipotezy pozwolą sformułować wiele uogólniających wniosków dotyczących użyteczności sekwencji przepływów pieniężnych jako sekwencji znaków tych przepływów, znaków i kwot przepływów oraz sekwencji wskaźników finansowych wyrażających komponenty rachunku przepływów pieniężnych. Przeprowadzone badanie prowadzi do wniosku, że sekwencja znaków oraz sekwencja kwot i znaków jest użyteczna w prognozowaniu upadłości jedynie pod pewnymi warunkami. Wskaźniki oparte na przepływach pieniężnych są słabyymi estymatorami bankructwa.

Słowa kluczowe: prognozowanie bankructwa, analiza przepływów pieniężnych, modelowanie bankructwa, modelowanie przepływów pieniężnych.